1. $Q = 2\min\{\frac{1}{2}k, L\}$; $r = 2$ and $w = 8$

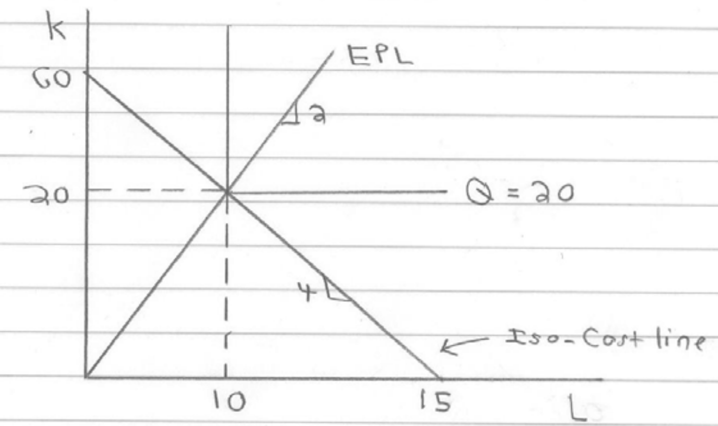
a) Each unit of output requires $\frac{2}{2} = 1$ unit of k and $\frac{1}{2}$ unit of L

Hence 20 units of output requires 20 units of k and 10 units of L $\boxed{k^* = 20,\ L^* = 10}$

b) Cost to produce 20 units of output :

$$\boxed{C = 20 \times 2 + 10 \times 8 = 120}$$

c)



D) $\boxed{\text{CRS}}$ Why? Double inputs and Output Doubles

$$\boxed{LRAC = \frac{120}{20} = 6}$$

2. A $MRTS_{L-K}$ is a constant and equal to 2

This implies that $\frac{MP_L}{MP_K} = 2 \Rightarrow MP_L = 2MP_K$

Also

(1) $16 = MP_L \times 4 + MP_K\, 8$ or

(2) $16 = (2MP_K) \times 4 + MP_K\, 8 = 16\, MP_K \Rightarrow MP_K = 1$
and $MP_L = 2$

Hence $\boxed{Q = 2L + 1K}$

Perfect Substitutes $\Rightarrow$ $\boxed{CRS}$

2B. a) Technology A: Each unit of output requires 1 unit of K and 2 L. Hence

$$\boxed{C^A(Q) = [1 \times 2 + 2 \cdot 1]\, Q = 4\, Q}$$

Technology B: $\frac{MP_K}{2} = \frac{\frac{1}{2}}{2} = \frac{1}{4} = \frac{1}{4} = \frac{MP_L}{1}$

Hence use any $\{K, L\}$ combination that produce desired output. Suppose use only L. Then

$\boxed{C^B(Q) = [4 \cdot 1]\, Q = 4\, Q}$ If Use Only K, same costs

b) <u>Yes.</u> If $w = 2$, Firm B can switch to all Capital in production and Costs do not change. But, Firm A's cost function changes to $C^A(Q) = [1 \cdot 2 + 2 \cdot 2]\, Q = 6Q$. Hence, Firm B now enjoys a cost advantage over firm A. This is an example of "<u>raising rivals' costs.</u>

Extra Credit

(1) $64 = K \cdot L \cdot R$ Also,

(2) $\frac{MP_K}{2} = \frac{MP_L}{2} = \frac{MP_R}{2} \Rightarrow$

(3) $\frac{LR}{2} = \frac{KR}{2} = \frac{KL}{2} \Rightarrow$

(4) $L = K = R$, so, upon substitution into (1),

(5) $64 = K^3 \Rightarrow K = 4 \Rightarrow L = R = 4$. Hence,

the cost of producing 64 units efficiently is given by

(6) $(4 \times 2) + (4 \times 2) + (4 \times 2) = \boxed{24 = C(64)}$